Modeling the dynamic properties of conventional and high-damping boring bars

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**Abstract**

Nowadays, the availability of reliable mathematical models of machining system dynamics is a key issue for achieving high quality standards in precision machining. Dynamic models can indeed be applied for tooling system design, preventive evaluation of cutting process stability and optimization of cutting parameters. This is of particular concern in internal turning, where the cutting process is greatly affected by the compliance of the tooling system. In this paper, an innovative hybrid dynamic model of the tooling system in internal turning, based on FE beams and empirical models, is presented. The model was based on physical and geometrical assumptions and it was refined by using experimental observations derived from modal testing of boring bars with different geometries and made of different materials, i.e. alloy steel and high-damping carbide. The predicted modal parameters of the tooling system (tool tip static compliance, natural frequency and damping coefficient of the dominant mode) are in good accordance with experimental values.

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**1. Introduction**

Dimensional accuracy and surface quality of high-precision mechanical parts produced by machining processes are greatly affected by the static and dynamic behavior of the machining system. Inappropriate machining system compliance or inadequate cutting parameters may give rise to undesired vibrational phenomena (chatter) arising during the cutting process, which is detrimental for surface finish and dimensional accuracy. Moreover, chatter may cause premature tool wear or tool breakage, and damage to machine tool elements such as tooling system or spindle bearings.

Machining system is composed of several interconnected elements, such as machine structure, machine tool drives, workpiece and workpiece fixtures, tooling system, spindle, etc. However, the dynamic relative compliance between the tool and the workpiece at the instantaneous point of contact is dominated by the most flexible elements of this kinematic chain.

In several important machining applications, such as internal turning or finish milling, the tooling system is the most critical component. Therefore, dynamic models of tooling systems have been extensively studied both in turning and milling, see Table 1.

Boring bar dynamic models proposed by Lazoglu et al. [1] and by Moetakef-Imani and Yussefian [2] were used for a realistic simulation of the cutting process. Other research works involving boring bar dynamic models aimed at developing chatter prediction methods [8,10,13] or effective chatter suppression strategies [3,4,12,14,15]. For instance, a special
boring bar endowed with a passive vibration absorber was modeled by Miguelez et al. in 2010 [3], while Mei et al.
developed a dynamic model including the effects of an advanced damping mechanism based on a magnetorheological fluid
[4]. Alternatively, a dynamic model of the tooling system can be applied for implementing active control techniques, as
that proposed by Pratt et al. in 2001 [15].

Essentially, three different approaches for modeling slender tooling systems can be found in literature. The first is
purely analytical, i.e., it is based on physical and structural equations governing the tooling system dynamic behavior [2,5].
The second is purely empirical: modal parameters of the machining system are measured by performing experimental
tests, without an explicit connection to the underlying dynamic structural equations [1,6]. The third is hybrid: a FE
analytical model of a relatively simple element of the spindle-holder-tool assembly is coupled with experimental transfer
functions describing the residual spindle-tool holder subsystem, in order to obtain the tool tip dynamic compliance, as
proposed by Park et al. in 2003 [7] and by Catania et al. in 2010 [8].

The dynamic features of slender tooling systems are usually derived from the Euler–Bernoulli beam model. However, as
illustrated by Budak et al. in 2006 [9], predictions obtained by applying Timoshenko beam theory [17] demonstrated a
better correlation to the numerical results obtained from sophisticate FE models and to experimental measurements
performed on real systems. A wider experimental validation of the proposed methodology was presented by the same
authors in 2007 [10], where they applied the developed model for a proper selection of design and operational parameters
regarding the spindle-holder-tool assembly.

Table 1
State of the art of tooling system dynamic modeling.

<table>
<thead>
<tr>
<th>Process</th>
<th>Tooling system geometry</th>
<th>Sensors for dynamic identification</th>
<th>Modeling approach</th>
<th>Beam model</th>
<th>Main purpose</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>$D \approx 25 \text{ mm}$ \newline $L/D \approx 7 \text{ mm}$</td>
<td>Microphone and dynamometer</td>
<td>Empirical</td>
<td>Harmonic oscillators</td>
<td>Simulation of cutting process dynamics</td>
<td>[1]</td>
</tr>
<tr>
<td>IT</td>
<td>$D = 20 \text{ mm}$ \newline $L/D = 7$</td>
<td>Impact tests equipment</td>
<td>Analytical</td>
<td>Euler–Bernoulli</td>
<td>Simulation of cutting process dynamics</td>
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<tr>
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<td>$D = 20 \text{ mm}$ \newline $L/D = 15$</td>
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<td>Euler–Bernoulli</td>
<td>Chatter suppression by passive device</td>
<td>[3]</td>
</tr>
<tr>
<td>IT</td>
<td>$D = 30 \text{ mm}$ \newline $L/D = 6$</td>
<td>Accelerometers</td>
<td>Analytical</td>
<td>Euler–Bernoulli</td>
<td>Chatter suppression by passive device</td>
<td>[4]</td>
</tr>
<tr>
<td>IT</td>
<td>$D = 40 \text{ mm}$ \newline $L/D = 5$</td>
<td>Accelerometers</td>
<td>Analytical</td>
<td>Euler–Bernoulli</td>
<td>Modeling boring bar dynamics</td>
<td>[5]</td>
</tr>
<tr>
<td>ET</td>
<td>$L = 49–59 \text{ mm}$</td>
<td>Strain gauges, Accelerometers</td>
<td>Empirical</td>
<td>Purely empirical</td>
<td>Statistical investigation of modal parameters</td>
<td>[6]</td>
</tr>
<tr>
<td>M</td>
<td>$D = 15–19 \text{ mm}$ \newline $L/D = 4–8$</td>
<td>Impact hammer, force sensors, accelerometer</td>
<td>Hybrid</td>
<td>Timoshenko</td>
<td>Modeling spindle-holder-tool assemblies</td>
<td>[7]</td>
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<tr>
<td>M</td>
<td>$D = 28–40 \text{ mm}$ \newline $L/D = 5–10$</td>
<td>Impact hammer, accelerometers</td>
<td>Hybrid</td>
<td>Harmonic oscillators</td>
<td>Chatter prediction</td>
<td>[8]</td>
</tr>
<tr>
<td>M</td>
<td>$D = 15 \text{ mm}$ \newline $L/D = 3.5$</td>
<td>Not specified</td>
<td>Analytical</td>
<td>Timoshenko</td>
<td>Modeling spindle-holder-tool assemblies</td>
<td>[9]</td>
</tr>
<tr>
<td>M</td>
<td>$D = 12 \text{ mm}$ \newline $L/D = 5–8$</td>
<td>Impact hammer, accelerometer</td>
<td>Analytical</td>
<td>Timoshenko</td>
<td>Chatter prediction</td>
<td>[10]</td>
</tr>
<tr>
<td>IT</td>
<td>$D = 40 \text{ mm}$ \newline $L/D &gt; 6$</td>
<td>Shaker, accelerometers</td>
<td>Analytical</td>
<td>Euler–Bernoulli</td>
<td>Modeling boring bar dynamics</td>
<td>[11]</td>
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<tr>
<td>ET</td>
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<td>Impact tests equipment</td>
<td>Hybrid</td>
<td>Harmonic oscillators</td>
<td>Chatter suppression by passive device</td>
<td>[12]</td>
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<tr>
<td>ET</td>
<td>$D = 70–120 \text{ mm}$ \newline $L/D = 2–7$ \newline (workpiece)</td>
<td>Accelerometers</td>
<td>Hybrid</td>
<td>Harmonic oscillators</td>
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<tr>
<td>IT</td>
<td>$L/D = 12$</td>
<td>Impact hammer, accelerometer</td>
<td>Analytical</td>
<td>Euler–Bernoulli</td>
<td>Increasing the boring bar dynamic stiffness</td>
<td>[14]</td>
</tr>
<tr>
<td>IT</td>
<td>$L/D = 12$ \newline $D = 25.4 \text{ mm}$</td>
<td>Impact tests equipment</td>
<td>Analytical</td>
<td>Harmonic oscillators</td>
<td>Chatter suppression by active control</td>
<td>[15]</td>
</tr>
<tr>
<td>IT</td>
<td>$L/D = 3–9$ \newline $D = 10–16$</td>
<td>Accelerometers</td>
<td>Hybrid</td>
<td>Timoshenko</td>
<td>Modeling boring bar dynamics</td>
<td>[16]</td>
</tr>
</tbody>
</table>

IT = internal turning; ET = external turning; M = milling.
In 2004 Andren et al. investigated the non-stationary stochastic behavior of boring bar vibrations and non-linear properties of its dynamic response [5]. In 2009, Akesson et al. discovered that boring bar dynamics were significantly affected by clamping procedure and by clamping conditions [11], which were considered as the main sources of non-linear dynamic properties.

Recently, the dynamics of conventional boring bars made of alloy steel were studied in a preliminary work by Totis et al. [16]. Specifically, the influence of boring bar geometry on the modal parameters was experimentally investigated. According to the presented conclusions, it was possible to model the tool tip static compliance and the main resonance frequency of the tooling system by applying a Timoshenko beam element coupled with toolholder dynamics. A simple empirical model was proposed for representing the damping coefficient behavior.

Nowadays, special boring bars made of high-damping materials are commercially available. They are particularly recommended for severe internal turning applications requiring high bar overhangs or when difficult to cut materials have to be machined. However, due to the considerably higher relative cost (more than twice that of standard boring bars), their application is sustainable only when standard boring bars do not guarantee a stable cutting process. In the perspective of preliminary evaluating boring bar performance, reliable and practical models describing the tooling system dynamic behavior are needed. Nevertheless, the dynamic properties of high-damping boring bars have not been adequately explored yet.

The aim of this work was to extend the applicability of the mathematical model proposed in [16] to the case of high-damping boring bars made of sintered carbides. For this purpose, both conventional and high-damping boring bars were compared in this research, by analyzing different bar diameters $D$ and aspect ratios $L/D$, see Fig. 1(a). The compliance of toolholder-boring bar interface was included in the model, since it did significantly affect model accuracy. Model uncertainties were finally discussed, in order to provide useful data for robust chatter prediction methods.

2. Mathematical modeling

Tooling system was composed of several mechanical components, including the toolholder, the boring bar and the cutting insert, see Fig. 2(a). The toolholder was further clamped at the machine tool head. In the perspective of applying FE models, some nodes were defined along the main axis of the tooling system. Specifically, node 3 was tool tip axial position, corresponding to the point of excitation for impact tests. Displacement of tooling system was measured at node 2, which was close but not equal to node 3. The ideal bar-toolholder interface was node 1, whereas node 0 was considered as the...
actual bar–toolholder interface. In other words, the total bar overhang was the sum of the nominal overhang \( L_{01} \) plus an additional overhang \( L_{0} \), the latter representing the imperfect constraint at node 1, in accordance with Andren et al. [18] and with Moetakef-Imani and Yussefian [2]. One key hypothesis of the proposed model is that

\[
\frac{L_{01}}{D} \approx \text{cost.} \tag{1}
\]

independently of bar material.

In general, tooling system dynamics can be expressed in the frequency domain by

\[
\begin{bmatrix}
    u_{3,x}(j\omega) \\
    u_{3,y}(j\omega)
\end{bmatrix} =
\begin{bmatrix}
    W_{u3F3,xx}(j\omega) & W_{u3F3,xy}(j\omega) \\
    W_{u3F3,xy}(j\omega) & W_{u3F3,yy}(j\omega)
\end{bmatrix}
\begin{bmatrix}
    F_{3,x}(j\omega) \\
    F_{3,y}(j\omega)
\end{bmatrix}
\tag{2}
\]

where \( W_{u3F3,xx} \) and \( W_{u3F3,yy} \) are the direct transfer functions and \( W_{u3F3,xy} \) and \( W_{u3F3,yx} \) are the cross transfer functions. However, the following simplifications

\[
W_{u3F3,xy} \approx W_{u3F3,yx} \approx 0 \tag{3}
\]

and

\[
W_{u3F3,xx} \approx W_{u3F3,yy} \tag{4}
\]

are commonly adopted in literature thanks to toolholder—boring bar axial symmetry, see for instance [21]. In order to verify these hypotheses, some preliminary impact tests were performed with different tooling system configurations. In general, cross transfer functions were negligible and direct transfer functions were very similar. However, for most practical purposes it is sufficient to model the dynamic behavior along the radial direction, as proposed by Lazoglu et al. [1]. Accordingly, tool tip dynamic behavior along the radial direction — simply denoted by \( W_{u3F3} \) — will be mainly considered in the following treatment.

Tool tip dynamic compliance in the radial direction can be modeled as a sum of vibration modes

\[
W_{u3F3}(j\omega) = \sum_{i} \frac{G_{i}}{(j\omega/\omega_{ni})^{2} + 2\zeta_{i}j\omega/\omega_{ni} + 1} \tag{5}
\]

whose modal parameters (mode residues \( G_{i} \), natural frequencies \( \omega_{ni} \) and damping coefficients \( \zeta_{i} \)) depend on boring bar configuration (bar geometry and material) and they may slightly change from radial to tangential direction. For many applications of practical interest it is sufficient to consider the following approximation

\[
W_{u3F3}(j\omega) \approx \frac{G_{u3F3}}{(j\omega/\omega_{n1})^{2} + 2\zeta_{1}j\omega/\omega_{n1} + 1} \tag{6}
\]

where \( G_{u3F3} \) is the total tool tip static compliance, while \( \omega_{n1} \) and \( \zeta_{1} \) are the natural pulsation and the damping coefficient associated to the main resonance peak, respectively. Therefore, mathematical models for estimating the afore said modal parameters were developed and calibrated by using experimental data.

2.1. Tool tip static compliance model—\( G_{u3F3} \)

Machine tool head—toolholder receptance at node 0, (see Fig. 2(c)) is approximated by the two degrees of freedom dynamic model

\[
\begin{bmatrix}
    K_{uu} & 0 \\
    0 & K_{qq}
\end{bmatrix}
\begin{bmatrix}
    u_{0} \\
    q_{0}
\end{bmatrix} =
\begin{bmatrix}
    F_{0} \\
    M_{0}
\end{bmatrix} \tag{7}
\]

where the off-diagonal terms were neglected for the sake of simplicity. Toolholder translational and rotational static compliances are, respectively,

\[
G_{uu0} = \frac{1}{K_{uu0}}, H_{qq0} = \frac{1}{K_{qq0}} \tag{8}
\]

These unknown coefficients have to be experimentally determined, under the key hypothesis that they are independent of boring bar geometry and material.

A Timoshenko beam element is chosen for modeling the cantilever boring bar from node 0 to node 3. Since the geometry of the beam is given, the only unknown is the effective Young Modulus \( E \), especially in the case of carbide boring bars. First guess values of the Young Modulus \( E_{th} \) for the considered bar materials are listed in Table 2.

In order to determine the effective Young Modulus \( E \), it is necessary to compare the experimental static compliance \( G_{exp} \) measured at node 2 with the theoretical prediction \( G_{u3F3} \). With this purpose in mind, let us temporarily set at zero both translation and rotation of node 0. Bar deflection at node 2 \( (u_{2}) \) caused by input force applied at tool tip \( (F_{3}) \) can be
derived from the Timoshenko beam element, as follows

$$\frac{EJ}{L_{02}(1+\eta)} \left[ \frac{12}{L_{02}^2} \frac{6}{(4+\eta)} \right] \begin{bmatrix} u_2 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} F_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} 1 \\ L_{23} \end{bmatrix} F_3$$

(9)

This yields

$$u_2 \big|_{(u_0,\varphi_0 = 0)} = \begin{bmatrix} \frac{L_{02}^3}{3EJ} + \eta \frac{L_{02}^2}{12EJ} + \frac{L_{23}^2}{2EJ} \\ G_{ext} \end{bmatrix} \begin{bmatrix} F_3 \end{bmatrix}$$

(10)

being

$$\eta = \frac{12EJ}{L_{02}^2AG_KK}$$

(11)

where $G_K$ and $K$ are the shear modulus and Timoshenko shear coefficient for the hollow circular cross section of the boring bar, whose area $A$ and moment of inertia $I$ are given by the well known relations

$$J = \frac{\pi(D^4-d^4)}{64}; \quad A = \frac{\pi(D^2-d^2)}{4}$$

(12)

Now, by imposing the continuity at node 0 and by considering the toolholder flexibility, the following expression is obtained

$$G_{exp} \approx G_{u2F3} = \frac{1}{E} \left( \frac{L_{02}^3}{J} \left( \frac{1}{3} + \frac{\eta}{12} \right) + \frac{L_{02}^2L_{23}}{2J} \right) + \frac{G_{u0} + H_{\varphi 0}(L_{03}L_{02})}{G_{exp}} \frac{1}{v_1}$$

(13)

The effective Young Modulus $E$ is expected to be close to the first guess value $E_{th}$, thus the deviation $\delta_E$ can be introduced, as follows

$$\frac{E_{th}}{E} = 1 + \delta_E, \text{ with } |\delta_E| \ll 1$$

(14)

Accordingly, Eq. (13) can be rewritten in the following form

$$\left( 1 - \frac{V_1}{G_{exp}E_{th}} \right) \approx \frac{V_1}{G_{exp}E_{th}} + \left( \frac{1}{G_{exp}} \right) G_{u0} + \left( \frac{L_{01}L_{02}}{G_{exp}} \right) H_{\varphi 0}$$

(15)

where the unknown parameters $\delta_E$, $G_{u0}$ and $H_{\varphi 0}$ can be estimated by linear regression. It has to be pointed out that the renormalization of Eq. (13) obtained by dividing both members of the equation by $G_{exp}$ was fundamental for achieving a good distribution of relative errors when applying regression.

After mechanistic estimation of the unknown parameters, tool tip static compliance can be computed as

$$G_{u3F3} = \frac{L_{02}^3}{J} \left( \frac{1}{3} + \frac{\eta}{12} \right) + G_{u0} + H_{\varphi 0}L_{03}^2$$

(16)

It has to be pointed out that static compliance at node 2 ($G_{u2F3}$) was significantly smaller than tool tip static compliance at node 3 ($G_{u3F3}$) because of the eigenvectors' shape (on average, it was 30% smaller). Therefore, it was fundamental to take into account the effects of $L_{23} > 0$ in the model, for a correct estimate of the unknown coefficients of Eq. (15).
2.2. Main natural frequency model—$f_{n1}$

Main natural frequency is derived by combining the toolholder dynamic receptance with the Timoshenko beam element representing the boring bar into one FE model with only four degrees of freedom, as follows

$$
\begin{bmatrix}
\dot{u}_0 \\
\dot{\varphi}_0 \\
u_3 \\
\dot{\varphi}_3
\end{bmatrix} + C_{03} \begin{bmatrix}
\dot{u}_0 \\
\dot{\varphi}_0 \\
u_3 \\
\dot{\varphi}_3
\end{bmatrix} + K_{03} \begin{bmatrix}
u_0 \\
\varphi_0 \\
u_3 \\
\varphi_3
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
F_3 \\
0
\end{bmatrix}
\Rightarrow W_{u_3f_3}(j\omega) = \frac{u_3(j\omega)}{F_3(j\omega)} \Rightarrow f_{n1,th}(\text{dominant mode})
\tag{17}
$$

In detail, the global stiffness matrix $K_{03}$ is obtained by including the toolholder stiffness matrix $K_0$ of Eq. (7) into the $4 \times 4$ stiffness matrix representing the Timoshenko beam. Similarly, mass matrix $M_{03}$ is obtained by including the $2 \times 2$ toolholder mass matrix $M_0$ into the $4 \times 4$ mass matrix of the Timoshenko beam. For the sake of simplicity, in the current case the toolholder mass matrix $M_0$ was approximated by a diagonal matrix with sufficiently high constant coefficients such as they generate low resonance frequencies which do not interfere with the first natural frequency of the cantilever boring bar. Finally, $C_{03}$ is the viscous damping matrix, which can be neglected when computing the natural frequencies since they only depend on mass and stiffness matrixes.

First guess values of Young Modulus $E_{th}$ and toolholder compliances $G_{oth}$ and $H_{oth}$ are used. However, it has to be noticed that the main natural frequency $f_{n1,th}$ is only slightly affected by $G_{oth}$ and $H_{oth}$, therefore such dependences could be neglected. An expression including the only important unknown is finally obtained

$$
\frac{1}{f_{n1,exp}} \approx \sqrt{1 + \delta_E} \frac{1}{f_{n1,th}} = \left( \frac{1}{f_{n1,exp}} \right) \frac{f_{n1,exp}}{2f_{n1,th}} \delta_E
\tag{18}
$$

where the last relation is valid provided that $\delta_E$ is small.

Main natural frequencies derived from Eq. (17) and experimental values can also be compared with the classical Euler–Bernoulli formula

$$
f_{n1,th} = \frac{v_1}{2\pi} \sqrt{\frac{E}{\rho A l^4_{03}}}
\tag{19}
$$

where $v_1 = 3.516$ for a cantilever beam.

Accordingly, for a (full) circular cross section main natural frequency can be roughly estimated by

$$
f_{n1,th} \approx 0.14 \left( \frac{L}{D} \right)^2 D^{-1} \sqrt{\frac{E}{\rho}}
\tag{20}
$$

2.3. Damping coefficient model—$\xi_1$

In general, it is very difficult to develop predictive physical models of damping phenomena regarding mechanical structures. This is due to the strong stochastic behavior exhibited by damped mechanical systems. In order to minimize model complexity and enhance model applicability, semi-empirical models are usually preferred to describe the damping coefficient behavior.

A well common damping model is that of Rayleigh [20], representing the damping matrix as a linear combination of mass and stiffness matrix, as follows

$$
\begin{bmatrix}
\varphi_0 \\
\varphi_3
\end{bmatrix} = \tau \begin{bmatrix}
1 \\
1
\end{bmatrix} + \psi \begin{bmatrix}
0 \\
0
\end{bmatrix}
\tag{21}
$$

where $\tau$ and $\psi$ are unknown coefficients that must be determined by comparison with experimental data. By expressing Eq. (17) through the eigenvectors base, it is possible to demonstrate that

$$
\xi_1 = \frac{1}{2} \left( \tau \frac{1}{f_{n1,th}} + \psi f_{n1,th} \right)
\tag{22}
$$

which can be used for estimating $\tau$ and $\psi$ by linear regression on the experimental values $\xi_{1,exp}$.

3. Experimental data analysis

3.1. Experimental set-up and procedures

In order to prove the applicability of the proposed models, impact tests were carried out. Input force $F_3$ was applied on tool tip by using an impact hammer Dytran type5800B4, with sensibility of 2.41 mV/N, connected to an amplifier Kistler type 5134B, see Fig. 2(a). Tooling system displacements were measured at positions 2 and 0, see Fig. 2(b), by means of two non contact eddy...
current probes Micro-Epsilon type ES1 (measuring range 1 mm, sensitivity ~ 12 mV/μm) connected to eddyNCDT 3010-M controllers. Regarding data acquisition, all sensor signals were sampled at 20 kHz by using a National Instruments device (cDAQ-9178 with NI9215 modules) connected via USB to a PC. Data were elaborated in MathWorks MATLAB environment.

Tooling system consisted of an HSK-63A spindle adapter (Sandvik 392.41027-63 25 090B) installed on the machine tool head of an OKUMA MULTIS B300W CN lathe. Four commercial boring bars of different geometry and materials were tested, as listed in Table 2. Intermediate elements were Sandvik 132L-2516-B (for \( D = 16 \) mm) and 132L-2510-B (for \( D = 10 \) mm).

Impact tests were performed with different boring bar overhang \( L \), i.e., with different aspect ratios \( L/D \), as reported in Table 3. Impact tests were executed by hammering the tool tip in the radial direction \( (y=p) \), since it is mostly responsible for the regenerative effect causing chatter. For the sake of comparison, some impact tests were also performed by hammering the tool tip in the tangential direction \( (x=c) \), confirming the hypotheses outlined in the previous section. Experiments along the radial direction will be discussed in the following.

### 3.2. Experimental static compliances and natural frequencies

An example of the Empirical Transfer Function Estimates \( W_{u2F3} \) and \( W_{u0F3} \) is given in Fig. 3(a). Experimental tool tip static compliance \( G_{\text{exp}} \approx G_{u2F3} \) was estimated by averaging the amplitude of \( W_{u2F3} \) in the low-frequency range. An estimate of the toolholder translational compliance at node 0 was also derived from \( W_{u0F3} \), i.e.,

\[
G_{\text{t0}} \approx \frac{1}{K_{\text{t0}}} G_{u0F3} \tag{23}
\]

On the other side, toolholder rotational compliance \( H_{\text{t0}} \) could neither be estimated from \( W_{u2F3} \) nor from \( W_{u0F3} \). By considering bar number 1 as a reference, the importance of the different terms composing the total static compliance was evaluated, see Fig. 3(b). It can be noticed that for \( L/D > 5.5 \), the Bernoulli term \( G_{\text{ber}} \) is dominant (more than 90% of the total compliance). For smaller \( L/D \) values other terms become relevant too, such as the translational toolholder flexibility (about 20% of the total compliance), the rotational toolholder flexibility (5%) and the correction due to Timoshenko theory (5%). However, the relative importance of the Timoshenko term is considerably higher than 5%, when neglecting toolholder flexibility.

Regarding the natural frequencies observed in Fig. 3(a), \( W_{u2F3} \) is characterized by one dominant peak at the natural frequency of the cantilever bar, and by other minor peaks which are approximately located in the range 500–2000 Hz. From the analysis of \( W_{u0F3} \), such minor peaks can be attributed to machine tool head—toolholder dynamics. This is confirmed in Fig. 4, where the ETFEs of the other tooling system configurations are illustrated. For each given bar type, some minor resonance frequencies can be identified in the range 500–2000 Hz, independently of \( L/D \). On the contrary, there is one relevant resonance peak whose natural frequency strongly depends on \( L/D \), in accordance with the explanation given above.

### Table 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Levels</th>
<th>Values</th>
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</thead>
<tbody>
<tr>
<td>Boring bar material</td>
<td>2</td>
<td>Steel, high-damping carbide</td>
</tr>
<tr>
<td>External diameter ( D )</td>
<td>2</td>
<td>10, 16 mm</td>
</tr>
<tr>
<td>Aspect ratio ( L/D )</td>
<td>~8</td>
<td>3 – 9</td>
</tr>
<tr>
<td>Replications</td>
<td>1 or 2</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 3.** Example of ETFEs measured close to tool tip (node 2) and to toolholder (node 0) (a). Importance of different terms for static compliance estimate (b).
Natural frequencies $f_{n1,\text{exp}}$ corresponding to the main resonance peaks of each $W_{i2F3}$ were used as experimental values for Eq. (18). In most cases, only one resonance peak was identified, deriving from pure cantilever behavior, while other peaks were negligible. This was particularly true for boring bars of small diameter ($D=10\,\text{mm}$). On the contrary, multiple resonance peaks were identified especially in the case of the high-damping boring bar with $D=16\,\text{mm}$, likely due to its higher stiffness.

In order to estimate the unknown Young Moduli and the toolholder static compliances, Eqs. (15) and (18) were at last combined into the following system of linear equations comprising both bar materials

\[
\begin{bmatrix}
Q_{G,\text{stl}} \\
Q_{G,\text{crb}} \\
Q_{f,\text{stl}} \\
Q_{f,\text{crb}}
\end{bmatrix}
= \begin{bmatrix}
V_{G,\text{stl}} & 0 & V_{u0,\text{stl}} & V_{\phi0,\text{stl}} \\
0 & V_{G,\text{crb}} & V_{u0,\text{crb}} & V_{\phi0,\text{crb}} \\
V_{f,\text{stl}} & 0 & 0 & 0 \\
0 & V_{f,\text{crb}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{\text{stl}} \\
\delta_{\text{crb}} \\
G_{\text{stl}} \\
H_{\phi\alpha}
\end{bmatrix}
\]

(24)

Fig. 4. ETFEs measured for normal steel boring bars (a) and (b) and high-damping carbide boring bar (c) and (d), against $L/D$ ratio.
where the subscript \( \text{stl} \) refers to conventional boring bars made of alloy steel, the subscript \( \text{crb} \) refers to high-damping boring bars made of carbide, and the \( O's \) are columns of zeros, whose length is equal to the number of experimental points available for each case.

For a fixed \( L_0/|D| \) ratio the system (24) was solved by applying linear regression. Afterwards, relative errors were computed as follows

\[
\begin{align*}
\varepsilon_{\text{rel,G}} &= \frac{G_{\text{mod}} - G_{\text{exp}}}{G_{\text{exp}}} \times 100 \\
\varepsilon_{\text{rel,fn}} &= \frac{f_{n1,\text{mod}} - f_{n1,\text{exp}}}{f_{n1,\text{exp}}} \times 100
\end{align*}
\]

where \( G_{\text{mod}} \) and \( f_{n1,\text{mod}} \) are the static compliance and the main natural frequency estimated through the obtained model, respectively. The calculation was repeated by varying \( L_0/|D| \) and by recalculating the relative errors. By observing the behavior of the mean \( \mu_{\text{rel}} \) and the standard deviation \( \sigma_{\text{rel}} \) of the relative errors against \( L_0/|D| \), an optimal solution was found, as reported in Table 4 and illustrated in Figs. 5 and 6.

It has to be pointed out that the choice expressed by Eq. (1) was preferred to the alternative \( L_0/|D| \) = cost, which resulted in a far worse model. Moreover, in order to test the influence of each unknown term of Eq. (24) on output variance, the stepwise regression method was applied [19]. According to this modeling and statistical tool, both Young Moduli \( \delta \)'s and toolholder compliance terms gave a significant contribution to output variance, justifying their presence in the model.

It can be noticed that the translational toolholder compliance \( G_{\text{stl}} \) reported in Table 4 is in good accordance with the theoretical values of Table 2.

As expected, tooling system static compliance increases when the aspect ratio \( L/|D| \) is increased or when considering smaller bar diameters \( D \). For a given bar geometry, high-damping boring bars have more than twice the stiffness of those made of conventional alloy steel, which is a direct consequence of the higher Young Modulus of sintered carbide.

Tool tip static compliances \( G_{\text{ntF3}} \) are predicted with good accuracy by the proposed model, as evidenced by the trends reported in Figs. 5 and 6 and by the statistics of relative errors listed in Table 4.

Experimental resonance frequencies increase when \( L/|D| \) is decreased, when considering smaller bar diameters \( D \), or when considering high-damping carbide boring bars (which are characterized by slightly greater values of \( E/\rho \)), in accordance with Eq. (20). Experimental resonance frequencies are nevertheless slightly more scattered around the predicted values, especially for boring bars of bigger diameter. The outliers highlighted in Fig. 6 are associated to relatively high resonance peaks characterizing the ETFEs, which cannot be accurately predicted by the model because of the adopted model simplifications. Accordingly, such outliers were excluded from the computations.

A deeper analysis of relative error distributions revealed small systematic errors affecting the high-damping boring bars when considered separately. Such bias are probably due to heterogeneous mechanical properties of the high-damping material caused by a stochastic manufacturing process.

Euler–Bernoulli model (Eq. (13) with \( \eta = 0 \) and Eq. (19)) was also tested. Optimal values of the unknown parameters were very similar to those obtained with the Timoshenko model, but relative errors distributions were slightly worse. However, Euler–Bernoulli model is sufficiently accurate, provided that \( L/|D| > 3 \).

### 3.3. Experimental damping coefficient

From each ETFE, experimental damping coefficient \( \xi_{1,\text{exp}} \) of the dominant mode was estimated as follows

\[
\xi_{1,\text{exp}} \approx \frac{G_{\text{ntF3}}}{2 \min(|\text{Im}(W_{\text{ntF3}})|)}
\]

By analyzing the trends of \( \xi_{1,\text{exp}} \) against the considered factors, a significant dependence on \( L/|D| \) and on bar material was found out, whereas no significant dependence on bar diameter was evidenced, see Fig. 7.

### Table 4

Estimated model coefficients and relative errors statistics.

| Model       | \( L_0/|D| \) [ ] | Bar material | \( E \) [GPa] | \( G_{\text{nt}} \) [\( \mu \text{m}/\text{Nm} \)] | \( H_{\text{nt}} \) [\( \mu \text{rad}/\text{Nm} \)] | \( \mu_{\text{rel,G}} \) [%] | \( \sigma_{\text{rel,G}} \) [%] | \( \mu_{\text{rel,fn1}} \) [%] | \( \sigma_{\text{rel,fn1}} \) [%] |
|-------------|-------------------|--------------|---------------|------------------|-------------------|------------------|------------------|------------------|------------------|
| Euler–Bernoulli | 0                 | Steel        | 144           | 0.062            | 2.38              | 2.9              | 7.6              | 22.6             | 16.7             |
|             |                   | Carbide      | 390           | 0.028            | 6.08              | -1.1             | 4.9              | 2.9              | 8.3              |
| Euler–Bernoulli | 0.7 (opt.)      | Steel        | 212           | 0.028            | 6.08              | -1.1             | 4.9              | 2.9              | 8.3              |
|             |                   | Carbide      | 558           | 0.023            | 6.45              | -2.3             | 5.2              | 0.2              | 8.1              |
| Timoshenko  | 0.7 (opt.)       | Steel        | 216           | 0.028            | 6.08              | -1.1             | 4.9              | 2.9              | 8.3              |
|             |                   | Carbide      | 563           | 0.023            | 6.45              | -2.3             | 5.2              | 0.2              | 8.1              |
Linear regression on experimental data was then performed according to the Rayleigh model of Eq. (22). Relative errors were computed according to the formula

$$
e_{\text{rel}, \log \xi} = \frac{\log(\xi_{1,\text{mod}}) - \log(\xi_{1,\text{exp}})}{\log(\xi_{1,\text{exp}})} \%$$

(27)

However, the resulting interpolation was not satisfactory, as proved by the statistics of relative errors reported in Table 5.

Accordingly, the following empirical model was proposed

$$\xi_{1,\text{exp}} \approx \tau (L/D)^\psi$$

(8)

The model is based on two unknown coefficients $\tau$ and $\psi$, the former depending on bar material while the latter being independent of bar material. On the average, damping coefficients were considerably higher (about doubled) in the case of...
high-damping boring bars, thanks to the higher intrinsic damping capabilities of sintered carbide materials with respect to alloy steels. This was also evidenced by the $\tau$ coefficients estimated for the two materials, see Table 5.

This second model demonstrated better interpolation capabilities than the Rayleigh model, as evidenced by relative error statistics listed in Table 5. Besides, it has to be noticed that a smaller number of unknown model coefficients was used (three instead of four).

Data dispersion around the predicted values is considerable. Nevertheless, it is very difficult to further improve the model due to the intrinsically unpredictable behavior of most damped mechanical system.

By using the estimated model coefficients it is now possible to obtain the approximate boring bar dynamic model of Eq. (5), for any given tooling system configuration. Also, a measure of model accuracy is now available thanks to the analysis of relative errors.

4. Conclusions

According to the considerations and experimental results presented in this work, we may draw the following conclusions.
In most experimental cases, boring bar dynamics measured at tool tip can be well approximated by a single harmonic oscillator whose modal parameters are the total static compliance $G_u$, the natural frequency $f_{n1}$ and damping coefficient $\xi_1$ of the dominant mode of vibration.

A dynamic model composed by a FE beam coupled with toolholder dynamics was proposed for estimating the total static compliance and the main natural frequency. In order to obtain good interpolation results, it was fundamental to consider an additional bar overhang for modeling the non-ideal constraint at the toolholder-boring bar interface. Specifically, best results were achieved by assuming that both the ratio of the additional boring bar overhang to bar diameter and the static compliance of the tooling system evaluated at toolholder-boring bar interface were constant, independently of boring bar material and geometry.

Young Modulus estimates of conventional alloy steel and high-damping carbide were in good accordance with expected values, confirming the considerably higher stiffness of the latter material (between twice and three times). However, the application of the Timoshenko beam element did not greatly improved data interpolation in comparison with the Euler–Bernoulli beam, provided that the effect of toolholder-boring bar interface was adequately taken into account.

In general, predicted static compliances were in good accordance with experimental values, as evidenced by the analysis of relative errors (bias $|\mu_{rel,G}| \leq 2.3\%$, standard deviation $\sigma_{rel,G} \leq 7.3\%$, for both bar materials).

On the other side, experimental resonance frequencies trends were generally in good accordance with theoretical expectations (for instance, they were slightly higher when considering high-damping carbide boring bars, for a fixed bar geometry).

Also, the estimate of the main resonance frequency provided by the model was satisfactory but slightly less accurate (bias $|\mu_{rel,f_{n1}}| \leq 1.6\%$, standard deviation $\sigma_{rel,f_{n1}} \leq 8.7\%$, for both bar materials). Estimates regarding boring bars of bigger diameter were affected by a greater discrepancy. This was due to the dynamic interaction between boring bar dynamics and spindle-tool holder dynamics, causing some unexpected resonance peaks which could not be predicted by the model.

Experimental damping values mainly depended on the $L/D$ ratio and on boring bar material. Also, they were considerably higher in the case of high-damping boring bars (about double), thanks to the higher intrinsic damping
properties of sintered carbide materials with respect to alloy steels. The combination of higher stiffness and more effective damping can explain the superior dynamic behavior of high-damping boring bars during machining.

Interpolation results obtained by applying the Rayleigh model were not satisfactory. A simpler empirical model based on a smaller number of model coefficients was finally proposed, demonstrating better interpolation capabilities (bias $|\mu_{rel,log}| \leq 3.2\%$, standard deviation $\sigma_{rel,log} \leq 20.3\%$ for both bar materials). However, it is very difficult to further improve the model due to the intrinsically stochastic nature of damped mechanical systems.

It would be of further interest to apply the proposed models for developing reliable and robust chatter prediction methods for internal turning applications, which may be useful for a proper choice of tooling system configuration.

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References